# Intel ISEF 2016: Research Plan

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**Project Title:** A Deterministic Approach to the Location, Trajectory, and Collision Prediction of Particles in a Closed Environment

**Rationale:** In mathematics, dynamical systems are ones in which a point (or particle) is moving, dependent on time. Such a system can be bounded (closed) or unbounded, linear or nonlinear, integrable or chaotic; there are several different types, each with unique properties. Unbounded and nonlinear systems tend to involve the use of differential equations to resolve, while bounded tend to be trickier and exhibit more chaotic (essentially, less predictable) behavior.

Bounded systems can be of any shape: rectangular, triangular, circular, combinations of these, (similar to how organic molecules are structures form carbon, hydrogen, and oxygen atoms), or of any other shape. For any such system, it is important to be able to understand, given the necessary initial conditions, where a particle will be situated after some time, the trajectory in which it travels, and whether or not two such particles will ever collide. It is also highly important to be able to characterize the extent of "chaos" in a system, as these problems are of major importance in the closely related fields of linear dynamical systems and chaos theory.

There exists much work in this area, for example the well-known *Alhazen's Billiards Problem*, which asks at what angle a billiard ball must be rebounded off the circumference of a circle in order to collide with another billiard ball inside the circle. More recent work seeks to understand more about the paths followed by these objects, for example, whether or not these are cyclic, limit cycles are present (i.e., the paths infinitely tend towards one central path), or how rebounds in a closed system impact the path followed. There is substantial work concerning the behavior of nonlinearly moving particles in open environments, while closed environments are not considered as often.

The main gap in the existing research into this problem, however, is that many techniques provide either *probabilistic* or *simulation-based* solutions. The former involve solutions in which only "likely" and "unlikely" are defined, as opposed to definitive answers to properties that are by nature, definitive. Simulation-based solutions – which are used in video games for collision detection even today – require repeated computer-based discrete-event testing, which is slow and generally cannot provide a complete solution. As such, it is important that a *deterministic* solution be found – one that would use finite-length algorithms to determine the sought properties of a particle in a linear, two-dimensional, bounded system. This is thus the aim of my project: to formulate a deterministic solution to the questions set out before, concerning the location and trajectory of a particle at any time, prediction of collision, and further understanding of the extent of chaos or stability in closed, two-dimensional environments.

**Research Questions:**

1. How can the position and trajectory of a particle in a two-dimensional environment be determined in a deterministic manner?
2. Given that some pairs of particles will necessarily collide while other pairs will never, how can this collision (or lack thereof) be mathematically substantiated?
3. How does forming a larger system from joined smaller basic shapes impact the properties of the system?
4. Can the Maximal Lyapunov Exponent be determined, well-approximated, or even simply better-described using these deterministic results?
5. Are there time-periods in a chaotic system when the movement of particles is non-chaotic (or stable), and can the set of these periods be determined?
6. Does mapping the distance between two particles with mildly different initial conditions give rise to unexpected results, such as fractal nature or parabolic curves?

**Procedures & Data Analysis:**

*Note: Several of the procedures stated here are generalized (but not ambiguous) due to the fact that many of the later procedures, for example in stages 4 and 5, depend almost wholly on the results of the earlier procedures, i.e. stages 1, 2, and 3. In addition, the specific mathematical concepts that will be used cannot be predicted at this stage. As a result, only an overview of each step has been provided. However, this encapsulates the essence of the research procedure that will be followed.*

1. Case 1: Rectangle
    a. Determine formulae/finite algorithms for calculating the position and trajectory of a particle at time $t$
    b. Determine formulae/finite algorithms for determining whether or not two particles will ever collide, and if so, at what time $t$
    c. Use linear transformations (matrices) to determine formulae/finite algorithms to apply (1a) and (1b) to a rectangle positioned anywhere in the $xy$-plane
2. Case 2: Triangle
    a. Determine formulae/finite algorithms for calculating the position and trajectory of a particle in a right-angled triangle at time $t$
    b. Determine formulae/finite algorithms for determining whether or not two particles will ever collide in a right-angled triangle, and if so, at what time $t$
    c. Use results from (2a) and (2b) to determine the same for non-right-angled triangles
    d. Use linear transformations (matrices) to determine formulae/finite algorithms to apply (2c) to a triangle positioned anywhere in the $xy$-plane
3. Case 3: Circle and Semicircle
    a. Determine formulae/finite algorithms for calculating the position and trajectory of a particle in a circle at time $t$
    b. Determine formulae/finite algorithms for determining whether or not two particles will ever collide in a circle, and if so, at what time $t$

    c. Use the results from (3a) and (3b) to interpolate the same for semicircles
4. Combined Systems
    a. Begin with an easier case: determine formulae/finite algorithms for calculating the position and trajectory of a particle within an environment composed of a rectangle with semicircles on either end
    b. Second case: the same as (4a), but for triangles on either end
    c. Use results from (4a) and (4b) to form a generalization, although this will require further work that cannot be predicted at this preliminary stage
5. Chaos and Stability of these Dynamical Systems
    a. Use the definition of a Lyapunov exponent to explore how paths of particles differ given mildly different initial conditions
    b. In the resulting particles separation graph, determine whether or not periodicity or fractal structures are a possibility
    c. Determine formulae/finite algorithms to determine the "stable space", which is the infinite set of time regions when the particles' movement is deterministic and wholly predictable

**Risk & Safety:**

No specific safety measures are to be followed for this project.

**Engineering Goals:**

*Note: as with the procedures, these goals are likely to change as the project proceeds, due to expectation of unexpected but interesting findings.*

The first goal of this project is to *determine formulae and/or algorithms to determine the position, trajectory, or predict the collision between two particles in two-dimensional environments*.

The second goal is to *use these results in better understanding the chaotic nature and extent of complex systems*, possibly through better approximating the nature of the associated Lyapunov exponents.

**Bibliography (for preliminary research only – in-depth research and acknowledgements not listed)**

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